Core Mathematics C2 Paper C

1. Giving your answers in terms of π, solve the equation

$$3 \tan^2 \theta - 1 = 0,$$

for θ in the interval $-\pi \leq \theta \leq \pi$. [5]

2. Given that $p = \log_2 3$ and $q = \log_2 5$, find expressions in terms of p and q for

(i) $\log_2 45$, [3]

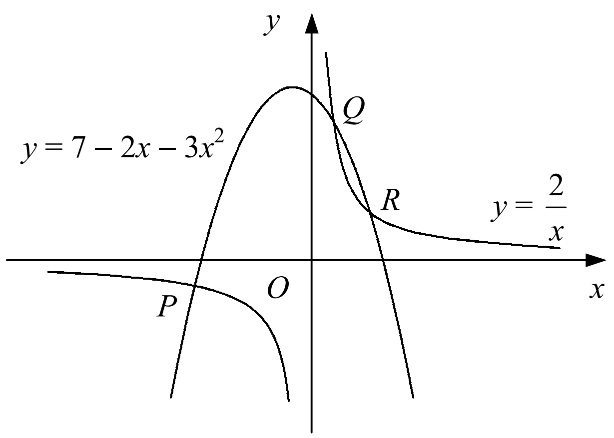
(ii) $\log_2 0.3$ [3]

3. For the binomial expansion in ascending powers of x of $(1 + \frac{1}{4}x)^n$, where n is an integer and $n \geq 2$,

(i) find and simplify the first three terms, [3]

(ii) find the value of n for which the coefficient of x is equal to the coefficient of x^2. [3]

4.



The diagram shows the curves with equations $y = 7 - 2x - 3x^2$ and $y = \frac{2}{x}$.

The two curves intersect at the points P, Q and R.

(i) Show that the x-coordinates of P, Q and R satisfy the equation

$$3x^3 + 2x^2 - 7x + 2 = 0.$$ [2]

Given that P has coordinates $(-2, -1)$,

(ii) find the coordinates of Q and R. [6]

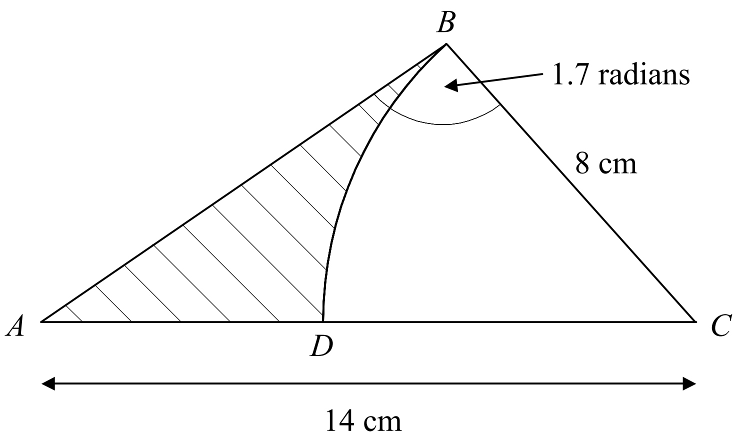
5. The curve $y = f(x)$ passes through the point $P(-1, 3)$ and is such that

$$\frac{dy}{dx} = -\frac{4}{x^3}, \quad x \neq 0.$$

(i) Find $f(x)$. [4]

(ii) Show that the area of the finite region bounded by the curve $y = f(x)$, the x-axis and the lines $x = 1$ and $x = 4$ is $4\frac{1}{2}$. [4]

6.



The diagram shows triangle ABC in which $AC = 14$ cm, $BC = 8$ cm and $\angle ABC = 1.7$ radians.

(i) Find the size of $\angle ACB$ in radians. [4]

The point D lies on AC such that BD is an arc of a circle, centre C.

(ii) Find the perimeter of the shaded region bounded by the arc BD and the straight lines AB and AD. [4]

7. *(a)* Given that $y = 3^x$, find expressions in terms of y for

(i) 3^{x+1}, [2]

(ii) 3^{2x-1}. [2]

(b) Hence, or otherwise, solve the equation

$$3^{x+1} - 3^{2x-1} = 6.$$ [5]

Turn over

8. *(i)* Given that

$$\int_1^3 (x^2 - 2x + k)\ \mathrm{d}x = 8\tfrac{2}{3},$$

find the value of the constant k. [6]

(ii) Evaluate

$$\int_2^\infty \frac{6}{x^{\frac{5}{2}}}\ \mathrm{d}x,$$

giving your answer in its simplest form. [5]

9. The second and fifth terms of a geometric series are –48 and 6 respectively.

(i) Find the first term and the common ratio of the series. [4]

(ii) Find the sum to infinity of the series. [2]

(iii) Show that the difference between the sum of the first n terms of the series and its sum to infinity is given by 2^{6-n}. [5]